Bhartiya Institute of Engineering and technology
Department of Civil Engineering
Theory of structures - II (6CE1A)
Semester - VIth MM-20

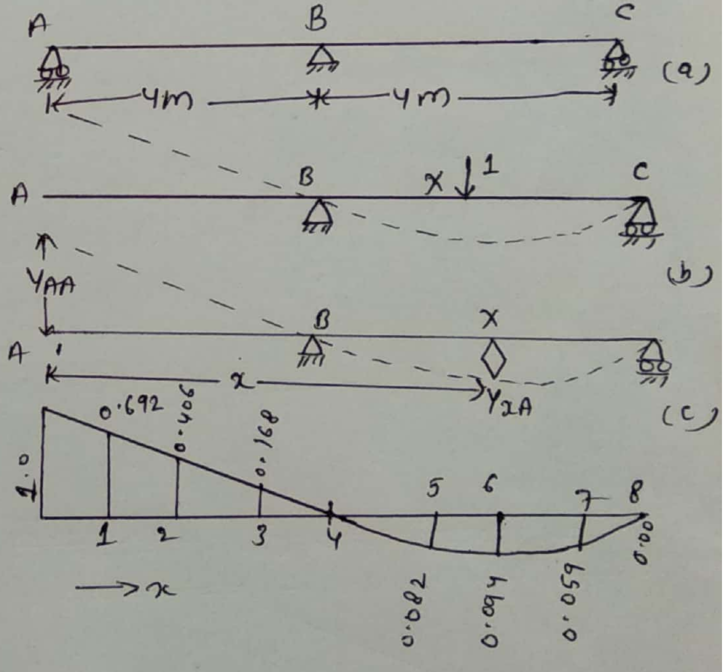
Q.1)(a) Determine the influence line for the continuous beam shown in fig. Compute the ordinates at every 1m interval.

(b) A symmetrical parabolic arch with a central hinge, of rise r and span L, is supported at its ends on pins at the same level. What is the value of the horizontal thrust when a load W which is uniformly distributed horizontally covers the whole span?

Ans.1)(a) Apply a unit vertical load at A, as shown in fig

EI Constant

$$R_A = \frac{y_{xA}}{y_{AA}}$$



from fig. C :-

$R_B = 2\downarrow$ and $R_C = 1\uparrow$

$$EI\frac{d^2y}{dx^2} = -M_x = -1\times x \,|+ R_B(x-4)$$

$$= -x \,| + 2(x-4)$$

$$EI\frac{dy}{dx} = \frac{-x^2}{2} + C_1 \,| + (x-4)^2$$

and

$$EIy = \frac{-x^3}{6} + C_1x + C_2 \,| + \frac{(x-4)^3}{3}$$

At $x = 4m,\ y = 0,$

$$\therefore 4C_1 + C_2 = \frac{64}{6}$$

At $x = 8m,\ y = 0,$

$$\therefore 8C_1 + C_2 = 64$$

$$\therefore C_1 = \frac{40}{3} \text{ and } C_2 = \frac{-128}{3}$$

Hence $EIy = \frac{-x^3}{6} + \frac{40}{3}x - \frac{128}{3} \Big| + \frac{(x-4)^3}{3}$

At $x = 0,\ y = y_{AA} = \frac{1}{EI}\left(-\frac{128}{3}\right) = \frac{128}{3EI}$

At $x = x,\ y = y_{xA} = \left[-\frac{x^3}{6} + \frac{40}{3}x - \frac{128}{3}\Big| + \frac{(x-4)^3}{3}\right]\frac{1}{EI}$

The values of y_{Ax} for various of x are given in table. The term $\frac{1}{EI}$ has been omitted for convenience.

x(m)	y_{xA}	$R_A = y_{xA}/y_{AA}$
0	$0 + 0 - 128/3$	1
1	$-\frac{1}{6} + \frac{40}{3} - \frac{128}{3} = -\frac{177}{6}$	+0.692
2	$-\frac{8}{6} + \frac{80}{3} - \frac{128}{3} = -\frac{104}{6}$	+0.406
3	$-\frac{27}{6} + \frac{120}{3} - \frac{128}{3} = -\frac{43}{6}$	+0.168
4	$-\frac{64}{6} + \frac{160}{3} - \frac{128}{3} = 0$	+0.00
5.	$-\frac{125}{6} + \frac{200}{3} - \frac{128}{3} + \frac{1}{3} = +\frac{21}{6}$	−0.082
6.	$-\frac{216}{6} + \frac{240}{3} - \frac{128}{3} + \frac{27}{3} = +\frac{15}{6}$	−0.094
7	$-\frac{343}{6} + \frac{280}{3} - \frac{128}{3} + \frac{27}{3} = +\frac{15}{6}$	−0.059
8.	$-\frac{512}{6} + \frac{320}{3} - \frac{128}{3} + \frac{64}{3} = 0$	0

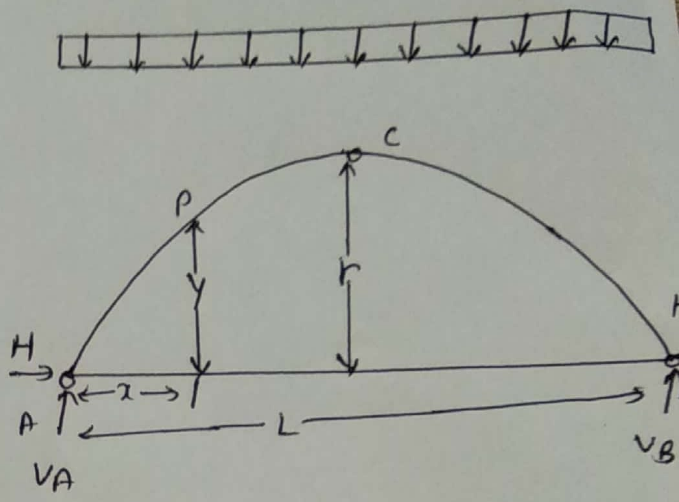
The I.L. for R_A is shown in fig. By Inspection, the plus sign shows that R_A acts upwards (↑) while minus sign shows that R_A acts downwords (↓)

Ans. 1) (b) The vertical reaction at A and B will be each equal to $\frac{W}{2}$.

for horizontal thrust, taking moment about C,

$$H \times r = \frac{W}{2} \times \frac{L}{2} - \frac{W}{2} \times \frac{L}{4} = \frac{WL}{8}$$

or $H = \frac{WL}{8r}$



Let us now consider any section at distance x from A.

~~Equating~~

Equation of the parabola is given by $y = \frac{4r}{L^2}x(L-x)$

$$\therefore M_x = H \cdot y + V_A \cdot x - \frac{Wx^2}{L2} = -\frac{WL}{8r} \cdot \frac{4r}{L^2} x(L-x) + \frac{W}{2}x - \frac{Wx^2}{2L}$$

$$= -\frac{Wx}{2} + \frac{Wx^2}{2L} + \frac{Wx}{2} - \frac{Wx^2}{2L} = 0$$

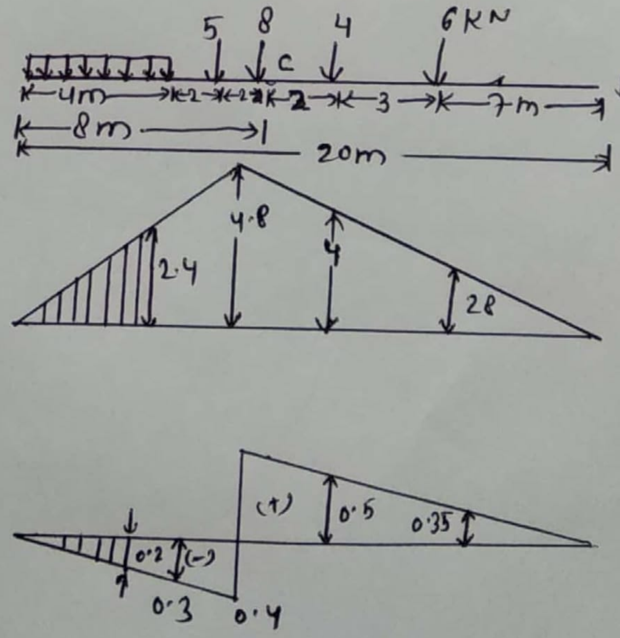
Q.2) (a) four wheel loads of 6, 4, 8 and 5 KN cross a girder of 20m span, from left to right followed by U.D.L. of 4KN/m and 4m long with the 6KN load leading. The spacing b/w the loads in the same order are 3m, 2m and 2m. The head of the U.D.L. is at 2m from the last 5KN load. Using influence lines, Calculate the S.F. and B.M. at a section 8m from the left support when the 4KN load is at center of the span.

(b) A parabolic arch hinged at the springings and crown has a span of 20m. The Central rise of the arch is 4m. It is loaded with a Uniformly distributed load of intensity 2KN/m on the left 8m length. Calculate a) the direction and ~~shear at 4m and 15m from the left end~~. magnitude of reactions at the hinges. b) the bending moment, normal thrust and shear at 4m and 15m from the left end. and c) maximum positive and negative bending moments.

Ans.2) a) Bending moment:

The ordinate of I.L. for B.M. at C $= \frac{8 \times 12}{20} = 4.8$

when the 4KN load is at the centre of beam, the arrangement of the other loads will be as shown in fig (a)



(a)

(b)

(c)

∴ ordinate under 6KN load
$= \frac{4.8}{12} \times 7 = 2.8$

Ordinate under 5KN load
$= \frac{4.8}{8} \times 6 = 3.6$

Ordinate under 4KN load
$= \frac{4.8}{12} \times 10 = 4$

ordinate under head of U.D.L. $= \frac{4.8}{8} \times 4 = 2.4$

$\therefore M_c = +\Sigma My = +[(6 \times 2.8) + (4 \times 4) + (8 \times 4.8) + (5 \times 3.6) + (\frac{1}{2} \times 2.4 \times 4 \times 4)]$

$M_c = +108.4\,kN\text{-}m$

(b) Shear force

The ordinates of I.L for S.F. at C are $-\frac{8}{20} = -\frac{2}{5} = -0.4$ and $+\frac{12}{20} = +\frac{3}{5} = +0.6$

Hence ordinate under 6 KN load $= \frac{0.6}{12} \times 7 = 0.35$

ordinate under 4KN load $= \frac{0.6}{12} \times 10 = 0.5$

ordinate under 5 KN load $= \frac{0.4}{8} \times 6 = 0.3$

ordinate under head of UDL $= \frac{0.4}{8} \times 4 = 0.2$

(i) maximum -ve S.F.,

for maximum -ve S.F., consider the 8KN load just to the left of C.

$\therefore F_c = -\{(8 \times 0.4) + (5 \times 0.3) + (\frac{1}{2} \times 4 \times 0.2 \times 4)\} + \{(4 \times 0.5) + (6 \times 0.35)\}$

$= -2.2\,KN$

(ii) maximum +ve S.F.

for maximum +ve S.F., consider the 8 KN load just to the right of C.

$\therefore F_c = +\{(6 \times 0.35) + (4 \times 0.5) + (8 \times 0.6)\} - \{(0.5 \times 0.3) + (\frac{1}{2} \times 4 \times 0.2 \times 4)\}$

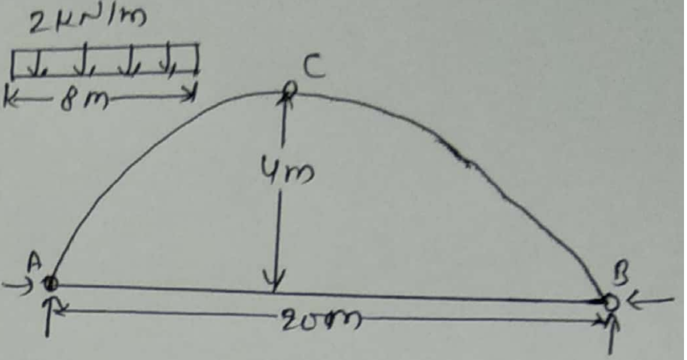
$= 5.8\,KN$

Hence the maximum S.F. at C, under the given load positions, will be the greater of the two, i.e. 5.8 KN.

Ans-2) (b)

a) Reactions at the hinges

for vertical reaction at A, take moment about B. Thus



$V_A \times 20 = 2 \times 8 \times (20-4)$

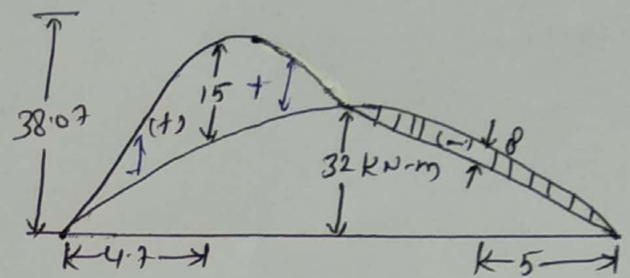
$V_A = 12.8\,kN$

Hence $V_B = 8 \times 2 - 12.8 = 3.2\,kN$

Since the bending moment at the hinge C is zero, we have

$M_C = (+3.2 \times 10) - H \times 4 = 0$

$\therefore \quad H = \frac{32.4}{4} = 8\,kN$



$\therefore$ Reaction at A

$= R_A = \sqrt{V_A^2 + H^2}$

$= \sqrt{(12.8)^2 + (8)^2} = 15.09\,kN$

Its inclination with the horizontal is given by,

$\tan\theta_A = \frac{V_A}{H} = \frac{12.8}{8} = 1.6; \quad \therefore \theta_A = 58°$

Rection at B $= R_B = \sqrt{V_B^2 + H^2} = \sqrt{(3.2)^2 + 8^2} = 8.62\,kN$

Its inclination with the horizontal is given by,

$\tan\theta_B = \frac{V_B}{H} = \frac{3.2}{8} = 0.4 \quad \therefore \theta_B = 21°48'$

The magnitude and dirn of the reaction at the crown will be same as that of the reaction at the hinge B, i.e., 8.62 kN at 21°48' with the horizontal. This is due to the fact that there is no loading between B and C. The reaction at B passes through C since $M_C = 0$.